B.Sc Part II

Infinite Series

Th A Series $\sum U_n$ converges iff to each $\epsilon > 0$, there exists a positive integer m such that

$$|U_{n+1} + U_{n+2} + \cdots + U_{n+p}| < \epsilon$$

$$\forall\ n \geq m \text{ and } p \geq 1$$

By Cauchy 'General Principle of Convergence (for Sequence). the Sequence (S_n) of partial sums of $\sum U_n$ converges iff to each $\epsilon > 0$ $\exists$ a positive integer m, such that

$$|S_{n+p} - S_n| < \epsilon\ \forall\ n \geq m \text{ and } p \geq 1$$

Or,
$$|U_{n+1} + U_{n+2} + \cdots + U_{n+p}| < \epsilon$$

$$\forall\ n \geq m \text{ and } p \geq 1$$

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Cauchy's Root test

If Σu_n is a positive term series such that $\lim_{n\to\infty} (u_n)^{1/n} = l$, then the series

1.) Converges if $l < 1$
ii.) diverges if $l > 1$ and
iii.) the test fails to give any definite information, if $l = 1$

Case I. $l < 1$

Let us select a positive number such that $l + \epsilon < 1$

Let $l + \epsilon = \alpha < 1$

Since $\lim_{n\to\infty} (u_n)^{1/n} = L$, therefore

$\exists$ a positive integer m such that

$$|(u_n)^{1/n} - l| < \epsilon \ \forall\ n \geq m$$

$$\Rightarrow l - \epsilon < (u_n)^{1/n} < l + \epsilon \ \forall\ n \geq m$$

$$\Rightarrow (l - \epsilon)^n < u_n < (l + \epsilon)^n$$

$$= \alpha^n \ \forall\ n \geq m \Rightarrow u_n < \alpha^n \ \forall\ n \geq m$$

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But Since $\sum \alpha^n$ is Cgt geometric Series (Common ratio $\alpha < 1$). therefore by comparision test the Series $\sum u_n$ Converges.

Case II $l > 1$

Let us select a positive number ϵ such that $l - \epsilon > 1$

Let $l - \epsilon = \beta > 1$

Since, $\lim_{n \to \infty} (u_n)^{1/n} = l$

therefore $\exists$ a positive integer m_1 Such that

$$l - \epsilon < (u_n)^{1/n} < l + \epsilon \quad \forall n \geq m_1$$

$$\Rightarrow (l-\epsilon)^n < u_n < (l-\epsilon)^n, \forall n \geq m_1$$

$$\Rightarrow u_n > (l-\epsilon)^n = \beta^n, \forall n \geq m_1$$

But Since $\sum \beta^n$ is a divergent geometric series (Common ratio $\beta > 1$), therefore by comparison test the series $\sum u_n$ diveges.